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The Pennsylvania State University

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**MULTI-OBJECTIVE, AUTO-OPTIMIZATION MODELING FOR RESOURCE  
ALLOCATION DECISION MAKING IN THE MILITARY HEALTH SYSTEM**

A Scholarly Paper in

Industrial Engineering and Operations Research

by

Nathaniel Drew Bastian, BS; MSc

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## **Abstract**

As health systems continue to grow with increasing demand for health services, the necessity to efficiently balance resources among hospitals is paramount. This paper explicates the structural similarities between multiple objective programming and data envelopment analysis in order to proffer an original, hybrid resource allocation-based optimization model that adjusts resources (system inputs) either with or without decision-maker input. The motivation for this study is to develop a decision-support model to be used by health care managers and policy-makers in support of resource allocations for large systems that are centrally controlled and funded, such as the Military Health System. In these systems, inputs are fixed at certain levels and may only be adjusted within Decision-Making Units (eg. medical treatment facilities). We provide a mathematical formulation and example solutions based on both textbook and real-world data. We also find utility in the use of multi-start evolutionary algorithms to store multiple optimal solutions for consideration by decision-makers. This multi-objective, auto-optimization model is currently being used for the performance-based analysis of U.S. Army hospitals.

**Key Words:** resource allocation, decision analysis, multiple objective programming, data envelopment analysis, health systems, military medicine

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## **Chapter 1. Introduction**

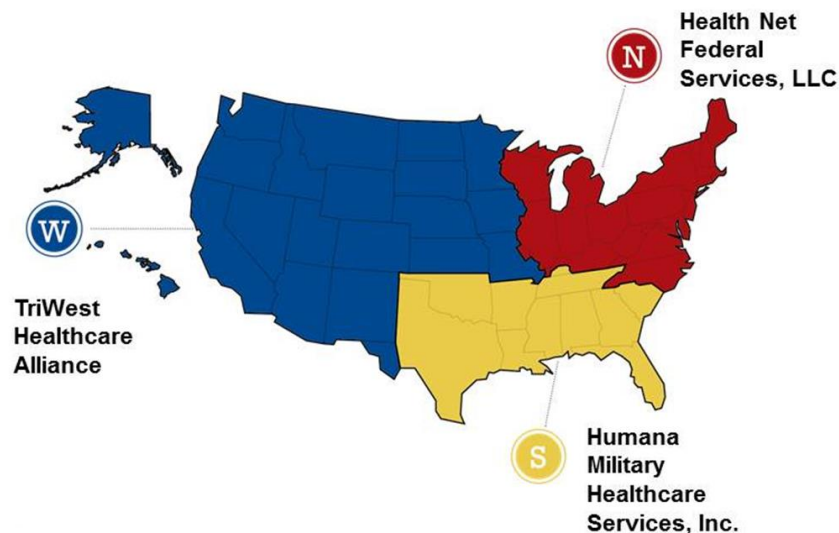
As our nation's inexorable growth in health care costs continues, the Department of Defense (DoD) health system, a large, centrally funded and operated health care system in the United States, is not immune to cost increases and growth. Due to the sheer size of this health system, it is imperative that we investigate efficiencies and determine new methods of analysis to maximize system performance by efficiently balancing costs, quality, satisfaction, and access across military hospitals. To do so requires careful management of major system components. The driving postulate of this paper is that key leaders of this health system wish to optimize inputs (specifically budgeted dollars and full-time equivalent healthcare providers) while maintaining outpatient weighted workload (relative value units or RVUs), inpatient weighted workload (relative weighted product or RWPs), prevention metrics, access metrics, and patient satisfaction. These components are intrinsically linked in a complex, multi-objective fashion. The motivation for this paper is then the necessity to efficiently balance resources among hospitals in large systems that are centrally controlled and funded while sustaining system output objectives.

### *1.1 The Military Health System*

The DoD Military Health System (MHS) is a complex organization with a \$52 billion budget that provides health services to over 4.5 million beneficiaries (uniformed service members, their family members, survivors and retirees) via the TRICARE program (MHS Stakeholders' Report, 2012). Within the MHS, there are more than 130,000 medical professionals (combining military and civilian) working in concert with the TRICARE network of providers. TRICARE combines the health care resources of the uniformed services with

networks of civilian health care entities to provide immediate access to high-quality health care services while maintaining the capability to support constant military operations (TRICARE, 2013).

TRICARE is available worldwide but operates out of four distinct geographic regions: North, South, West, and Overseas. In each geographic region, TRICARE partners with a regional contractor to provide health care services, administrative support and medical resources beyond what's available at Medical Treatment Facilities (MTF) and clinics. Currently, the TRICARE contractors include Health Net Federal Services, LLC (North Region), Humana Military Healthcare Services, Inc. (South Region), TriWest Healthcare Alliance (West Region), and International SOS (Overseas Region). The TRICARE regions based in the United States are depicted in Figure 1 below (current as of Fiscal Year 2013):



**Figure 1: TRICARE Stateside Regions**

TRICARE offers several different health plan options to meet the needs of authorized beneficiaries, but the two primary options include TRICARE Prime and TRICARE Standard and Extra (TRICARE, 2013).

TRICARE Prime is a military health plan in which the authorized beneficiary enrolls to receive health either within a MTF or in a participating civilian network component. This plan is similar to a Health Maintenance Organization (HMO) plan in that the enrollee agrees to receive their medical care from an assigned primary care manager (PCM). The enrollee must first seek care from the PCM for all non-urgent needs, and the PCM facilitates referrals to specialists within the civilian health care network. All active-duty military personnel are required to enroll in TRICARE Prime, and all other authorized beneficiaries other than the Medicare eligible may enroll. There are no restrictions regarding pre-existing conditions. TRICARE Prime provides comprehensive health coverage including: emergency care, outpatient visits, preventive care, hospitalization, maternity care, mental/behavioral health, and prescriptions. TRICARE Prime enrollees are guaranteed access within time limits. Drive time to the primary care site should not exceed 30 minutes. Waiting times for acute care may not exceed one day, while waiting time for routine care should not exceed one week. Specialty care is to be available within a one-hour drive with a maximum wait of four weeks for an appointment. Emergency care is to be available at all times. Active-duty military personnel and their families have no out-of-pocket costs for any type of care as long as care is received from the PCM or with a referral. All other enrollees are subject to annual enrollment fees of \$269.28 per individual or \$538.56 per family (for Fiscal Year 2013), varying copayments, and point-of-service charges (non-referred care) (TRICARE, 2013).

TRICARE Standard and Extra is a fee-for-service military health plan available to all non-active duty beneficiaries requiring no enrollment, which is equivalent to a traditional Preferred Provider Organization (PPO) health plan. TRICARE Standard and Extra provides the same comprehensive health coverage as TRICARE Prime, but the patient receives care from any

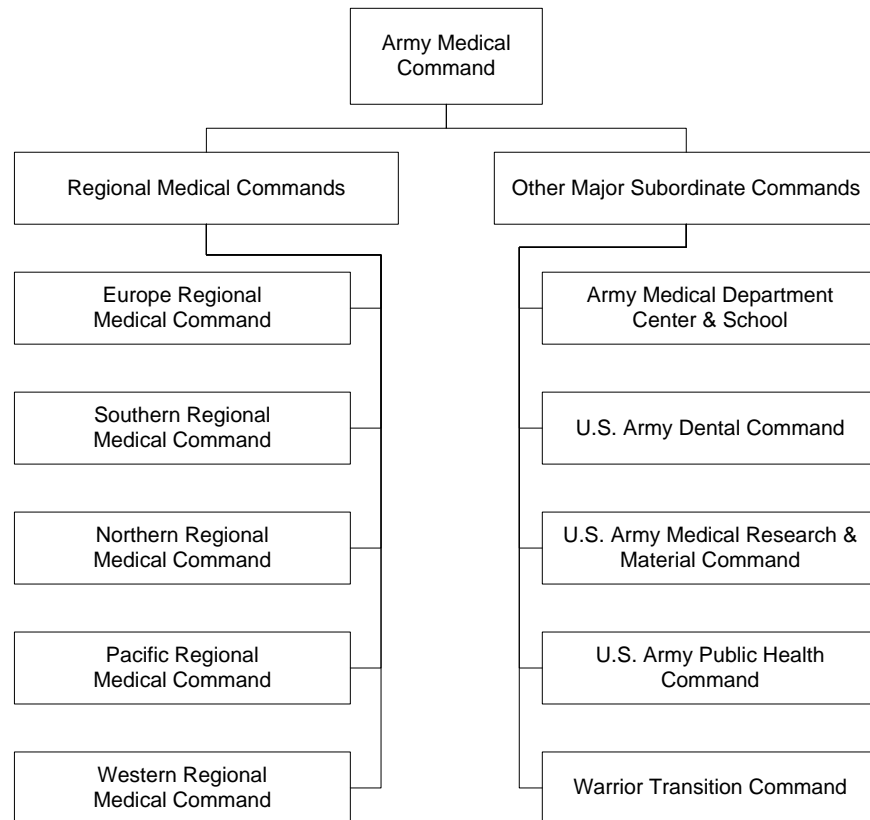
TRICARE-authorized provider, network or non-network rather than a PCM. Unlike the zero out-of-pocket costs associated with TRICARE Care, the TRICARE Standard and Extra costs vary depending on the sponsor's military status (active-duty vs. retired). For active-duty sponsors, the annual outpatient deductible is \$50 per individual or \$100 per family (Rank E-4 and below) or \$150 per individual or \$300 per family (Rank E-5 and above). For retired sponsors, the annual outpatient deductible is \$150 per individual or \$300 per family. In addition to the required annual outpatient deductible, patients are responsible for paying a cost share (percentage) based on the type of care and type of provider seen (network vs. non-network). Unlike network providers (Extra Option), non-network providers (Standard Option) may charge up to 15% above the TRICARE allowable charge and do not necessarily file health care claims on behalf of the patient. For example, the cost share of an emergency room visit for an active-duty family member is 15% of the negotiated rate under the Extra Option but 20% of the allowable charge under the Standard Option. For all other beneficiaries (eg. retirees), the cost share of an emergency room visit is 20% of the negotiated rate under the Extra Option but 25% of the allowable charge under the Standard Option. These inpatient cost shares differ by the type of care and are subject to change each fiscal year (TRICARE, 2013).

With a better understanding of the MHS, we can now focus on one of the major components of the MHS, the Army Health System.

### *1.2 The Army Health System*

The Army Health System (AHS) is the United States Army component of the MHS, which is responsible for the operational management of the Army Medical Department (AMEDD) health service support and force health protection missions. The AMEDD is a multi-billion dollar component of the United States Army responsible for providing efficient, effective

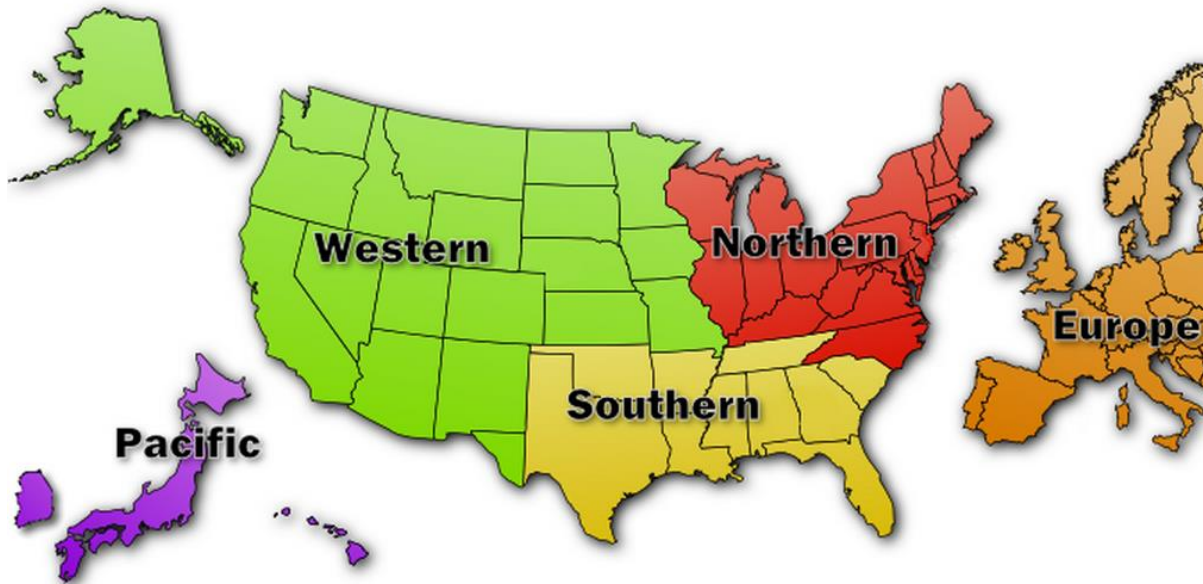
health care delivery to authorized beneficiaries across a continuum of both peacetime and wartime environments. All fixed hospitals (both inside and outside the United States) are commanded by the Army Medical Command (MEDCOM), which currently manages more than a \$13 billion budget and cares for more than 3.95 million beneficiaries (Army Medicine, 2012). The MEDCOM is composed of several subordinate organizations as depicted in Figure 2:



**Figure 2: MEDCOM Structure**

Each of the Regional Medical Commands (RMC) contain subordinate MTFs such as Army Medical Centers (MEDCEN), Army Community Hospitals (ACHs), Army Health Centers (AHCs), and other entities. In total, there are eight AMCs, 27 Medical Department Activities (MEDDAC) and numerous clinics in the United States, Europe, Korea and Japan. The map

depicted in Figure 3 shows the regional breakdown of RMCs, where the AMEDD has major MTFs (Army Medicine, 2012):



**Figure 3:** Geographic Breakdown of RMCs

When comparing Figure 1 and Figure 3, a complexity associated with the AMEDD structure is that the RMCs overlap multiple TRICARE coverage regions, serving as a potential source of inefficiency within the MHS.

In addition to the RMCs, the Other Major Subordinate Commands support the remaining functions of AMEDD's daily health care delivery mission. Medical research is unified under the U.S. Army Medical Research and Materiel Command (USAMRMC), which includes six research laboratories and five other commands that focus on medical materiel advanced development, strategic and operational medical logistics, and medical research and development contracting. Dental activities (DENTAC) are grouped under the U.S. Army Dental Command (DENCOM), which is organized into five regions called Regional Dental Commands (RDCs). The AMEDD Center & School (AMEDDC&S) is where the Army trains medical personnel, and



also serves as a think tank of military medicine, with a mission to envision, design and train a premier military medical force for full-spectrum operations. The Warrior Transition Command (WTC) serves as the central comprehensive source for warrior care support policy across the Army. Preventive medicine, health promotion, and veterinary services at the central, regional, and district level are grouped under the U.S. Army Public Health Command (USPHC).

With a better understanding of the complexity of the AHS, we can now review the literature on efficiency measurement techniques for resource allocation decision-making.

### *1.3 Literature Review*

In this paper, efficiency is defined as the ratio between health services delivered (outputs) and resources provided (inputs) at each MTF. Specifically, we consider technical (or frontier) efficiency that measures deviations in performance from that of best practice entities on the efficient frontier. Data Envelopment Analysis (DEA) is a deterministic, non-parametric linear programming (LP) technique developed by Charnes, Cooper, and Rhodes (1978) from the work of Farrell (1957). In DEA, a group of similar entities (e.g. military hospitals) are referred to as Decision Making Units (DMUs) which convert inputs into outputs. Using a LP model, DEA determines the optimal weights for each input and output that is most beneficial to an individual DMU. Upon determination of these optimal weights, the DMUs' ratio measures (weighted outputs to inputs) are compared to decide which DMUs are most efficient, where efficiency is based on the distance of their ratio measure from the piecewise-linear convex frontier created by the most efficient DMUs. Unlike regression analyses, DEA requires no explicit identification of underlying relations between inputs and outputs, and weights are not assigned *a priori*.

Another method of efficiency measurement is known as Stochastic Frontier Analysis (SFA), which was first introduced by Meeusen & Van den Broek (1977) and Aigner et al.

(1977). SFA is a stochastic, parametric econometric approach for production frontier modeling that accounts for random noise (error) affecting the production process. SFA specifies a production function and an error term composed of two parts with different statistical distributions (randomness and inefficiency). The usual assumption with the two-part error term is that the randomness (statistical noise) follows a normal distribution and the inefficiencies follow a half-normal, exponential, or truncated distribution. The random error term encompasses all events outside the control of the entity, such as the econometric errors and uncontrollable factors.

Many studies have applied efficiency measurement techniques in the civilian healthcare system (Hollingsworth, 2003; Worthington, 2004; Hollingsworth, 2008; Moshiri et al., 2010; Wilson et al., 2012), but here we will only examine those studies where they are applied in the military healthcare system. Charnes et al. (1985) were the first to use data envelopment analysis (DEA) in their evaluation of the performance of 24 Army health care facilities. They selected traditional workload criteria for analysis of outputs including personnel trained, relative weighted product (RWP), and clinic visits, which are considered traditional elements of production in health care. For inputs in their DEA model, they evaluated full time equivalent (FTE) employees by specific category, inpatient expenditures, outpatient expenditures, weighted procedures, occupied bed days, and operation room hours.

Following this first study, Mihara (1990) used both DEA and ordinary least squares (OLS) regression methods to conduct an efficiency analysis of the utilization of personnel at Navy MTFs for resource allocation decisions. The results of the study were used to baseline both physician requirements (workload and beneficiary dependent) and professional staff requirements (physician dependent). His study was limited in that the analyses were driven by only raw workload metrics, as other measures such as personnel readiness, prevention and

training were not included. Ozcan & Bannick (1994) conducted a longitudinal study of 124 DoD hospitals to evaluate trends in hospital efficiency based on data from the American Hospital Association Survey. This study was conducted at the strategic-level with little actionable information. Coppola (2003) used DEA to evaluate 78 MTFs using data from 1998 to 2002. For model input variables, he used costs, number of beds in the MTF, FTEs, and number of services offered. For model output variables, he used surgical visits, ambulatory patient visits (APV), emergency room visits, case mix adjusted discharges (CMAD), RWP and live births. As a limitation of his analyses, Coppola focused on workload as the primary measure for efficiency rather than the standardized outpatient workload metric called relative value units (RVUs), which captures the complexity of workload by accounting for resource consumption.

Fulton (2005) conducted an efficiency analysis of 24 Army community hospitals and medical centers from 2001-2003 using DEA, SFA and corrected ordinary least squares (COLS) models. He also analyzed hospital cost, which was modeled as a function of workload, population, a quality and prevention proxy, an access proxy, efficiency scores (and interactions), medical center status, and the interaction between medical center status and workload. Using both cross-sectional and panel series studies, Fulton applied various regression methods and estimation techniques to the models for comparison. From these analyses, he found that linear models with DEA efficiency provided better estimates than SFA models. Piner (2006) used DEA to evaluate clinical efficiency of 49 obstetric clinics at various MTFs and found that there was significant variability in the level of staffing and expenses among the clinics; this variability suggested inconsistencies in management across the clinics. The Army performed the highest in terms of average efficiency score, followed by the Air Force and then the Navy. Piner also

compared the size of the MTFs, which revealed that larger hospitals were more efficient than smaller hospitals.

Fulton et al. (2007) developed decision-support tools for performance-based resource allocation. Specifically, they used DEA and SFA to illustrate the feasibility of incorporating technical efficiency considerations in the funding of Army hospitals and identified the primary cost drivers for Army hospital operations. Using a three-variable, logarithmic-linear model, they found that \$120 million could be re-allocated to improve Army hospital performance. Schmacker & McKay (2008) used SFA to examine factors affecting the productive efficiency of primary care clinics in the MHS from 1999 through 2003. The following factors were specified as part of the inefficiency error component of the SFA model and were estimated simultaneously in the production function: staffing mix, use of physician-extenders, beneficiary mix, provider mix, military service branch, facility type, region, and year. From their analyses, they found that the primary care clinics associated with medical centers had significantly higher levels of productive efficiency. They also found that having proportionately more civilian healthcare staff had a positive impact on productive efficiency. Fulton et al. (2008) investigated military hospital cost models that incorporated quality, access and efficiency to provide decision-support for resource forecasting in the MHS. In their analyses, they used OLS regression estimation, ridge regression and robust regression methods to evaluate logarithmic-linear cost models that included DEA efficiency scores. They demonstrated that military hospital resource allocation models should include quality and efficiency as components along with the traditional elements of complexity-weighted inpatient and outpatient workload.

## *1.4 Overview and Structure*

In this chapter, we provided the necessary background information on the Military Health System, TRICARE, Army Health System, and Army Medical Department. We also reviewed the literature on efficiency measurement techniques (DEA and SFA) applied to problems in the military health system.

In Chapter 2, we first discuss multi-objective programming (MOOP) given the nature of the specified health care problem, as well as further describe DEA. After detailing these two methods, we follow Romero (1995), Joro, Korhonen & Wallenius (1998), and Korhonen & Syrjanen (2004) to show that they are nearly structurally identical and are related to distance models. After describing MOOP and DEA, we then discuss how these formulations evaluate rather than recommend for cases where inputs must remain at fixed levels. In other words, we show that evaluation of slack and reduced costs do not provide sufficient re-allocation decision support. In the case of the MHS, decision-makers seek to re-balance funding (cost) and personnel across the system. Doing so requires decision support regarding “winners” and “losers.”

In Chapter 3, we propose a mathematical programming model that seeks to optimize a system rather than evaluate a single DMU. Such a program is useful (vital) for health systems with fixed budgets and personnel authorizations (such as the MHS). While non-linearity in the model is inescapable, we demonstrate in Chapter 4 the formulation's effectiveness in solving a constant returns to scale (CRS, also known as CCR for Charnes, Cooper, and Rhodes) textbook healthcare problem as well as a variable returns to scale (VRS, also known as BCC for Banker, Charnes, and Cooper) real-world allocation problem associated with the Military Health System. Finally, we provide concluding remarks in Chapter 5.

## Chapter 2. Material and Methods

In this section, we thoroughly discuss multi-objective programming, data envelopment analysis, and the relationship between them in the context of the health care resource allocation problem. We also demonstrate how both methods are limited with regards to fixed input systems.

### 2.1 Multi-Objective Programming

The motivating example in the MHS consists of multiple competing objectives. While one might explore methods for modifying cost and/or production functions (e.g., Cobb-Douglas), the search here is restricted to the field of optimization and begins with a discussion of the basic linear program (LP).

The typical linear program may be expressed in matrix notation as follows.

$$\begin{aligned} & \text{Max } \vec{c}\vec{x} \\ & \text{Subject to } \begin{aligned} & A\vec{x} \leq \vec{b} \\ & \vec{x} \geq \vec{0} \end{aligned} \end{aligned} \tag{1}$$

Here, the objective function is composed of the  $1 \times n$  coefficient vector  $\vec{c}$  and the  $n \times 1$  decision variables  $\vec{x}$ . The constraint set is composed of the  $m \times n$  constraint coefficient matrix  $A$  along with our decision variables and the  $m \times 1$  right hand side constraints  $\vec{b}$ . Two objections to LP formulation are that linearity is often an over simplification of reality and that decision-makers are rarely concerned with just one objective function, as in our motivating example (French, 1984). To address non-linearity, one might modify the formulation as follows.

$$\begin{aligned} & \text{Max } g(\vec{x}) \\ & \text{Subject to } \vec{x} \in X = \{\vec{x} \mid A\vec{x} \leq \vec{b}, \vec{x} \geq \vec{0}\} \end{aligned} \tag{2}$$



This formulation provides a non-linear function  $g^*$ , which accounts for real-world complexity (and adds the same complexity to the solution algorithm). Still, the model does not consider multiple objective functions. The multi-objective optimization follows.

$$\begin{aligned} & \text{V-Max } \vec{g}(\vec{x}) \\ & \text{Subject to } \vec{x} \in X = \{\vec{x} \mid A\vec{x} \leq \vec{b}, \vec{x} \geq \vec{0}\} \end{aligned} \tag{3}$$

Here,  $\vec{g}^*$  is a set of functions that define all objectives to be maximized (or minimized). The problem with this formulation is that conflicting goals may prevent the simultaneous optimization of all the objectives. Generally, a Pareto optimal (efficient) solution set is sought such that for  $\vec{y} \in X$  there exists no  $g_i(\vec{y}) \geq g_i(\vec{x}) \forall \vec{x} \in X$  with strict inequality holding for at least one value of  $i$ . Here,  $\vec{y}$  is considered *dominated* or *inefficient*. (We will further discuss the notion of dominance later). Unfortunately, finding the Pareto optimal set still does not resolve the fundamental problem: which member of the efficient set does the decision-maker choose? To answer this question, one might consider a value function as in the following formulation (Cohen, 1978).

$$\begin{aligned} & \text{V-Max } f(\vec{g}(\vec{x})) \\ & \text{Subject to } \vec{x} \in X = \{\vec{x} \mid A\vec{x} \leq \vec{b}, \vec{x} \geq \vec{0}\}. \end{aligned} \tag{4}$$

The value function  $f^*$  is intended to monotonically increase with the decision-maker's preference. Alternatively, the decision-maker may just explore efficient sets.

Returning to the motivating example, it is assumed that the senior military medical decision-makers seek to minimize inputs while maintaining outputs constant. With an appropriately sufficient set of decision variables, one could attempt to devise a value function (as

in  $f^*$ ) above) based upon leaders' estimations of the importance of each item. In the case of this example, one might formulate the following set for optimization.

$$\begin{aligned} & \text{V-Max } f(g_i(\bar{x})) \\ & \text{Subject to } \bar{x} \in X = \{\bar{x} \mid A\bar{x} \leq \bar{b}, \bar{x} \geq \bar{0}\} \end{aligned} \tag{5}$$

The objective function here (assuming linearity) is an  $i \times i$  matrix coupled with an  $i \times 1$  set of decision variables for  $x$ . The constraint matrix  $A$  is of size  $m \times i$ .

Next, assume that the  $f$  functions are ordered in a monotonic increasing fashion by preference. That is,  $f(g_1^*)$  is less important to the decision-maker than  $f(g_2^*)$  and so on. If this importance function is discrete, the referent-derived weighting system is similar to that of utility matrix weights. The inherent assumption is that the weighting system is developed consistently, i.e., that the decision-maker makes choices consistently in accordance with the value function  $f$ .

If one makes an assumption that  $f$  and  $g$  are linear, then the formulation is called the Multiple Objective Linear Program (MOLP) and looks familiar.

$$\begin{aligned} & \text{V-Max } \bar{v} = C\bar{x} \\ & \text{Subject to } \bar{x} \in X = \{\bar{x} \mid A\bar{x} \leq \bar{b}, \bar{x} \geq \bar{0}\}, \end{aligned} \tag{6}$$

where  $\bar{x} \in R^n$ ,  $\bar{b} \in R^m$ , the constraints matrix  $A \in R^{m \times n}$  is of full rank  $m$ , and the objective function matrix  $C \in R^{1 \times n}$ . In (6),  $\bar{x}^* \in X$  is an *efficient solution* (or Pareto optimal) if and only if there does not exist another  $\bar{x} \in X$  such that  $C\bar{x} \geq C\bar{x}^*$  and  $C\bar{x} \neq C\bar{x}^*$ . Also,  $\bar{x}^* \in X$  is *weakly efficient* if and only if there does not exist another  $\bar{x} \in X$  such that  $C\bar{x} > C\bar{x}^*$ . An *inefficient* solution is one that is neither efficient nor weakly efficient. Let  $V = \{\bar{v} = C\bar{x} \mid \bar{x} \in X\}$  be the set of *feasible* objective function vectors. The vectors  $\bar{v} \in V$  that correspond to efficient solutions are

known as *non-dominated* criterion vectors, the vectors  $\bar{v} \in V$  that correspond to weakly efficient solutions are known as *weakly non-dominated* criterion vectors, and the vectors  $\bar{v} \in V$  that correspond to inefficient solutions are known as *dominated* criterion vectors (Joro et al., 1998).

Assume that there is interest in searching the non-dominated set of solutions. To do so with a linear  $f$  and  $g$  involves the projection of any point onto the set of non-dominated solutions. Wierzbicki (1980) provides an achievement scalarizing function (ASF), which is capable of this projection given a feasible or infeasible starting point. The ASF will be discussed later, after an investigation of the structurally-related mathematical programming technique of DEA.

## 2.2 Data Envelopment Analysis

DEA is a set of flexible, mathematical programming approaches for the assessment of efficiency, where efficiency is often defined as a linear combination of the weighted outputs divided by a linear combination of the weighted inputs as in the Charnes, Cooper, and Rhodes (CCR) model (Charnes et al., 1978), which is a constant returns to scale (CRS) formulation. Assume that an organization wishes to assess the relative efficiencies of some set of comparable subunits. (The subunits are called Decision Making Units or DMUs.) For each DMU, there is a vector of associated inputs and outputs of managerial interest (Cooper et al., 2007). In this case, the manager is interested in either maximizing the outputs while not exceeding current levels of inputs (output oriented) or minimizing the inputs without reducing any of the outputs (input oriented). Using our military hospital example, the inputs are budget, health care provider FTEs, and enrollment population, while the outputs are inpatient and outpatient weighted workload, a prevention metric, an access-to-care metric, and a patient satisfaction metric. In the case of DEA, the manager assumes that the traditional definition of engineering efficiency (ratio of weighted outputs to weighted inputs) will result in an acceptable solution for technical

efficiency. With these assumptions in place, one may formulate the following fractional programming problem that may be solved to determine technical efficiency, defined (for now) as the ratio of weighted outputs to weighted inputs, for each separate DMU (Wierzbicki, 1980). The following is known as the input-oriented CCR model:

$$\text{Max } \theta = \frac{\bar{u}^T \bar{y}_o}{\bar{v}^T \bar{x}_o} \quad (7)$$

Subject to:

$$\begin{aligned} \frac{\bar{u}^T \bar{y}_z}{\bar{v}^T \bar{x}_z} &\leq 1, \forall z \\ \bar{u} &\geq \bar{0} \\ \bar{v} &\geq \bar{0} \end{aligned}$$

In this formulation, there is a vector of outputs ( $\bar{y}$ ), a vector of inputs ( $\bar{x}$ ), and  $z$  DMUs. Efficiency is designated as  $\theta$ . The index  $o$  identifies the selected DMU for which an efficiency score will be generated. This mathematical program is run  $z$  times (the total number of DMUs), once to determine the efficiency of each DMU. (While MOLP simultaneously solves multiple objective functions given a value function, DEA optimizes efficiency for an individual DMU.) The components of the vectors  $\bar{u}$  and  $\bar{v}$  are the weights to be determined for the outputs and inputs, respectively. This model defines efficiency for the selected DMU as the weighted linear combination of its outputs divided by the weighted linear combination of its inputs, subject to the constraint that, for each DMU (including the one whose index  $z$  is  $o$ ), the efficiency cannot exceed one. All weights are restricted to be non-negative. This formulation is non-linear; however, if one seeks to maximize the outputs while maintaining inputs constant, it is trivial to normalize the weighted inputs such that they equal one.

$$\bar{v}^T \bar{x}_o = 1 \quad (8)$$

Multiplying the numerator and denominator of the objective function as well as constraint (7) and finishing by adding (8) to the constraint set yields the following formulation.

$$\text{Max } \theta = \vec{u}^T \vec{y}_o \quad (9)$$

Subject to:

$$\begin{aligned} \vec{u}^T \vec{y}_z - \vec{v}^T \vec{x}_z &\leq \bar{0}, \forall z \\ \vec{v}^T \vec{x}_o &= 1 \\ \vec{u} &\geq \bar{0} \\ \vec{v} &\geq \bar{0} \end{aligned}$$

For consistency with much (but not all) of the literature, this formulation in (9) is considered the dual, so taking the “dual of the dual” provides the primal. (The primal allows for better comparison with multiple objective programming.) In standard form, the primal follows.

$$\text{Min } \theta - \varepsilon(\bar{1}^T \vec{s}^+ - \bar{1}^T \vec{s}^-) \quad (10)$$

Subject to:

$$\begin{aligned} X\vec{\lambda} - \theta \vec{x}_o + \vec{s}^- &= 0 \\ Y\vec{\lambda} - \vec{y}_o - \vec{s}^+ &= 0 \\ \vec{\lambda}, \vec{s}^-, \vec{s}^+ &\geq \bar{0}, \varepsilon > 0 \end{aligned}$$

Here, the  $\varepsilon$  in the objective function is called the non-Archimedean element. This allows a minimization over efficiency score  $\theta$  to preempt the optimization of slacks  $(\vec{s}^-, \vec{s}^+)$ , which reflect output shortages and input excesses. A DMU that has an efficiency score of one and a zero-slack solution (for all slacks) is considered technically efficient or Pareto-Koopmans efficient. As defined in Cooper et al. (2007), Pareto-Koopmans efficiency is attained only if it is impossible to improve any input or output without worsening some other input or output. In all

other cases, it is possible to improve one or more of the inputs or outputs without worsening any other input or output.

Returning to our motivating example, one notes that the formulation of the DEA model will provide efficiency scores and slack information. The importance of any objective function is allowed to be a function of automatically generated weights. If a decision-maker deems quality is more important than access, then the above formulation does not provide a weighting system (e.g., the  $f$  function). Note that the weight of an objective should depend on the level of achievement, or otherwise a model may make solution tradeoffs that are inconsistent with the desires of the decision-maker.

Fortunately, there exist a variety of DEA based linear programs that assign weights to inputs and outputs based on importance of items. For example, Cooper et al. (2007) provide a weighted slacks-based model (W-SBM) with decision-maker weights applied. This model is similar to goal programming and is provided below (in fractional form) for reference.

$$\begin{aligned} \text{Min } \rho = & \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{w_i s_i^-}{x_{io}}}{1 + \frac{1}{t} \sum_{r=1}^t \frac{w_r s_r^+}{y_{ro}}} \\ & \sum_{i=1}^m w_i = m, \sum_{r=1}^s w_r = t \end{aligned} \tag{11}$$

Here, the  $w$  are weights and the  $s$  are slacks (input excesses or output shortages). The inputs belong to  $X$  and the outputs belong to  $Y$ . Returning to our example, one can readily see the capability of a decision maker to provide weights for budget and personnel inputs (index  $i$ ) as well as cost, quality, and access outputs (index  $r$ ). See Cooper et al. (2007, p. 105) for a complete discussion.



### 2.3 The Relationship Between DEA and MOLP

With this basic foundation in place, it becomes necessary to demonstrate how DEA and MOLP are structural twins. Doing so proves useful in formulating and solving the multi-objective, auto-optimization problem. The specific investigatory question of this next short section is how DEA relates to MOLP, and how one might leverage it to help the decision-maker balance competing objective functions automatically.

Joro et al. (1998) illustrated that DEA and MOLP are structurally related, as each might be formulated similar to the output-oriented CCR primal model. One first notes the need to restrict MOLP to solutions existing within the set of non-dominated criterion vectors through the use of an achievement scalarizing function (ASF), a function that projects any feasible or infeasible point onto the dominated set. Unlike Joro et al. (1998), we will show how DEA and MOLP are structurally related in terms of formulation of the input-oriented CCR primal model.

Consider the following formulation of an ASF provided by Wierzbicki (1980) which follows from (6), where  $r$  is the ASF:

$$Min(r) = \min \left\{ \max_{i \in P} \left( \frac{(z_i - v_i)}{\beta_i} \right) + \rho \sum_{i=1}^P (z_i - v_i) \right\} \quad (12)$$

$$\text{Subject to: } \bar{v} \in V = \{ \bar{v} = C\bar{x} \mid \bar{x} \in X, A\bar{x} \leq \bar{b}, \bar{x} \geq 0 \},$$

where,

$\bar{\beta} > 0, \bar{\beta} \in R^P \equiv$  vector of weights for each objective function

$\rho > 0 \equiv$  a non - Archimedean scalar element

$i \in P, P \equiv$  the set of objective functions

$\bar{z} \equiv$  vector of aspiration levels for the objective functions

$\bar{v} \equiv$  vector of current feasible objective function solutions

$c \in C, C \equiv$  matrix of objective function coefficients

Quite simply, one seeks to find  $\bar{x}$  that minimizes the largest deviation between the aspiration location (in objective function space) and our current location (in objective function space), while ensuring that the “slacks” for all vectors are as small as possible. This simplistic explanation provides the basis for the formulation of the ASF.

Following the procedures of Joro et al. (1998), we can further simplify our objective function with a simple replacement.

$$\min(r) = \min \left\{ \varepsilon + \rho \sum_{i=1}^P (z_i - v_i) \right\} \quad (13)$$

Subject to :

$$\bar{x} \in X, \bar{x} \geq \bar{0}$$

$$\varepsilon \geq \left( \frac{z_i - c_i \bar{x}}{\beta_i} \right), i = 1, 2, \dots, p,$$

where  $c_i$  ( $i = 1, 2, \dots, p$ ) refers to the  $i$ th row of the objective function matrix  $C$ .

We can also see that:

$$\begin{aligned} \beta_i \varepsilon \geq z_i - c_i \bar{x} &\Rightarrow C\bar{x} + \varepsilon \vec{\beta} \geq \vec{z} \Rightarrow C\bar{x} + \varepsilon \vec{\beta} - \vec{d} = \vec{z} \Rightarrow \beta_i \varepsilon - d_i = (z_i - c_i \bar{x}) \quad \bar{x}, \vec{d} \geq \bar{0} \\ A\bar{x} \leq \vec{b} &\Rightarrow A\bar{x} - \vec{b} - \vec{y}_o = \vec{y}_o \Rightarrow A\bar{x} - \vec{s}^+ = \vec{y}_o, \quad \bar{x} \geq \bar{0} \end{aligned}$$

One should also note the following, which derives directly from the objective function of (13):

$$\begin{aligned} \varepsilon + \rho \sum_{i=1}^P (z_i - v_i) &\Rightarrow \varepsilon + \rho \sum_{i=1}^P (z_i - c_i \bar{x}) \Rightarrow \varepsilon + \rho \sum_{i=1}^P (\beta_i \varepsilon - d_i) \Rightarrow \\ &\Rightarrow \varepsilon + \rho(\varepsilon \bar{1}^T \vec{\beta} - \bar{1}^T \vec{d}) = \varepsilon(\bar{1} + \rho \bar{1}^T \vec{\beta}) - \rho \bar{1}^T \vec{d} \end{aligned} \quad (14)$$

Since  $\rho \bar{1}^T \vec{\beta}$  is a constant, it may be removed from the minimization, leaving  $\varepsilon - \rho \bar{1}^T \vec{d}$ .

Letting  $C = X, A = Y, \bar{x} = \vec{\lambda}, \varepsilon = \theta, -\vec{\beta} = \vec{x}_o, -\vec{d} = \vec{s}^-$ , the reformulated reference point model

results in (15), which is structurally similar to that of the input-oriented CCR primal model:

$$\text{Min } r = \theta + \rho \bar{\mathbf{1}}^T \bar{\mathbf{s}}^- \quad (15)$$

Subject to:

$$X\bar{\lambda} - \theta \bar{\mathbf{x}}_o + \bar{\mathbf{s}}^- = \bar{\mathbf{z}}$$

$$Y\bar{\lambda} - \bar{\mathbf{s}}^+ = \bar{\mathbf{y}}_o$$

$$\bar{\lambda}, \bar{\mathbf{s}}^+, \bar{\mathbf{s}}^- \geq \bar{\mathbf{0}}, \rho > 0$$

In fact, placing the models side-by-side reveals few structural differences (modified from Joro et al., 1998).

DEA Input-Oriented CCR Primal Model	Reformulated Reference Point Model
$\text{Min } r = \theta - \varepsilon(\bar{\mathbf{1}}^T \bar{\mathbf{s}}^+ + \bar{\mathbf{1}}^T \bar{\mathbf{s}}^-)$	$\text{Min } r = \theta + \rho \bar{\mathbf{1}}^T \bar{\mathbf{s}}^-$
Subject to:	Subject to:
$X\bar{\lambda} - \theta \bar{\mathbf{x}}_o + \bar{\mathbf{s}}^- = \bar{\mathbf{0}}$	$X\bar{\lambda} - \theta \bar{\mathbf{x}}_o + \bar{\mathbf{s}}^- = \bar{\mathbf{z}}$
$Y\bar{\lambda} - \bar{\mathbf{y}}_o - \bar{\mathbf{s}}^+ = \bar{\mathbf{0}}$	$Y\bar{\lambda} - \bar{\mathbf{y}}_o - \bar{\mathbf{s}}^+ = \bar{\mathbf{0}}$
$\bar{\lambda}, \bar{\mathbf{s}}^-, \bar{\mathbf{s}}^+ \geq \bar{\mathbf{0}}, \varepsilon > 0$	$\bar{\lambda}, \bar{\mathbf{s}}^+, \bar{\mathbf{s}}^- \geq \bar{\mathbf{0}}, \rho > 0$

Given the obvious relationship between MOLP and DEA, we can see that it is largely a matter of preference and simplicity regarding selection of the base formulation for a multi-objective problem. Note that MOLP often ignores a very common trait of decision-making. That is, the attractiveness of any solution depends not only on the objectives but also on the level of achievement. As stated earlier, the weight of an objective should depend on the level of achievement, or else a model may make solution tradeoffs that are inconsistent with the wishes of the decision-maker. Over a narrow range, one can ignore this. In our comparison, we assume that over the range of possible outcomes, the marginal rate of substitution between objectives is constant. We now turn to an example of MOLP, DEA, and our hybrid model for discussion.

## 2.4 Motivating Example Comparison

Let us return to the example regarding military hospitals in the MHS. The components of the hospitals' activities are intrinsically linked. A possible multiple objective formulation related to six components might be the following.

$$\begin{aligned}
 & \left. \begin{aligned}
 & \text{Max } f(g(x1/x2, x3, x4, x5, x6)) \\
 & \text{Max } f(g(x2/x1, x3, x4, x5, x6)) \\
 & \text{Max } f(g(x3/x1, x2, x4, x5, x6)) \\
 & \text{Max } f(-g(x4/x1, x2, x3, x5, x6)) \\
 & \text{Max } f(-g(x5/x1, x2, x3, x4, x6)) \\
 & \text{Max } f(-g(x6/x1, x2, x3, x4, x5))
 \end{aligned} \right\} \rightarrow \text{Max } f \begin{bmatrix} g(x1 | x2, x3, x4, x5, x6) \\ g(x2/x1, x3, x4, x5, x6) \\ g(x3/x1, x2, x4, x5, x6) \\ -g(x4 | x1, x2, x3, x5, x6) \\ -g(x5 | x1, x2, x3, x4, x6) \\ -g(x6/x1, x2, x3, x4, x5) \end{bmatrix} \quad (16) \\
 & \text{st :} \\
 & \bar{x} \in X \mid \{A\bar{x} \leq \bar{b}, \bar{x} \geq \bar{0}\}
 \end{aligned}$$

This model seeks to maximize outputs while minimizing inputs subject to an  $f$  function determined by decision-makers. Alternatively, the original DEA input-oriented CCR primal model might be employed with a  $Y$  matrix consisting of output measures and an  $X$  matrix consisting of input measures. Of course, the DEA formulation generally seeks weights independent of a referent function; however, one could program the referent function as a series of constraints, as in the W-SBM model.

## 2.5 Limitations of MOLP and DEA for Fixed Input Systems

Decision-makers generally seek to investigate how inputs and outputs might be adjusted to improve the objectives. Certainly, system slack informs how one might be able to reduce inputs (for output-oriented models), but how should that slack be reallocated? The dual variables show the effects of a unit relaxation in constraint (e.g., inputs) might have on the objective function, although inputs in these large systems are largely fixed. Further, sensitivity analysis

associated with adjustments in any portion of the model can help inform decision-makers. But no amount of traditional manipulation of multiple objective optimization programs provides decision-makers clear recommendations about how to reallocate system inputs to achieve the highest level of satisfaction possible (efficiency). In fixed input systems such as the MHS, it becomes necessary to improve system performance by reallocating inputs among the existing DMUs. A multi-objective model that adjusted resources automatically across all MTFs to achieve maximum system efficiency would (at a minimum) provide decision-support and insight for leaders interested in evaluating multiple objectives simultaneously.

In the next chapter, we provide an alternative, multi-objective formulation that is based on a super-objective applied to traditional DEA analysis. This model assumes that the decision-maker would like to change inputs and outputs in order to have the resources necessary to achieve at least a minimum level of performance. We specify the formulation of the model, and provide a CRS textbook example as well as a VRS real-world example. We use three different solvers to investigate solutions to the formulation. A discussion of the basic model follows.

### **Chapter 3. Multi-Objective Auto-Optimization Model**

In the preceding chapter, we demonstrated that DEA and MOLP are related methods for evaluating efficiency in multiple objective problems. In DEA, we noted that the weights are often determined via optimization, while in MOLP, these weights are generally assigned. We also demonstrated that use of either approach does not provide sufficient information for optimizing system inputs over a system of systems with fixed inputs. We therefore propose a Multi-Objective Auto-Optimization Model (MAOM) for specific cases where one seeks to balance system components that might be interpreted as a performance ratio (not necessarily efficiency) as in the motivating example. Such a formulation should be able to identify inputs

that might be manipulated to improve system performance over multiple outputs (objectives). Essentially, this formulation should be able to provide sensitivity analysis to advise decision-makers how to optimally reallocate resources in order to attain the most efficient system possible. The next formulation applies to multiple objective problems that involve fixed inputs (or possibly outputs) that are fixed but can vary between DMUs. For example, we use a fixed budget large hospital organization (such as the MHS) that may reallocate resources among its facilities. A description of the model, its derivation, and an application follow. The definition of variables, sets, and data matrices follows.

### Indices

$o \equiv$  index of all  $m$  DMUs(hospitals)  
 $j \equiv$  index for outputs  
 $k \equiv$  index for inputs

### Decision Variables

$\partial_{ko} \equiv$  adjustments to each input  $k$  by DMU  $o$  with  $\partial \in \Delta$   
 $\alpha_{jo} \equiv$  weight for output  $j$  and DMU  $o$  with  $\alpha \in A$   
 $\lambda_{ko} \equiv$  weight for input  $k$  and DMU  $o$  with  $\lambda \in \Lambda$   
 $r \equiv$  lower limit for efficiency score required for all DMUs

### Data

$x_{ko} \equiv$  input  $k$  for DMU  $o$  with  $x \in X$   
 $y_{jo} \equiv$  output  $j$  for DMU  $o$  with  $y \in Y$

$$\text{Max } z = \sum_o \sum_j \alpha_{jo} y_{jo} \quad (17)$$

Subject to:

$$r \leq \sum_j \alpha_{jo} y_{jo}, \forall o \quad (18)$$

$$\sum_j \alpha_{jo=v} y_{jo} - \sum_k \lambda_{ko=v} (x_{ko} + \delta_{ko}) \leq 0, \forall o, v \in \{1, 2, \dots, N\} \quad (19)$$



$$\sum_k \lambda_{ko} (x_{ko} + \delta_{ko}) = 1, \forall o \quad (20)$$

$$x_{ko} + \delta_{ko} \geq 0, \forall k, o \quad (21)$$

$$\sum_o \delta_{ko} = 0, \forall k \quad (22)$$

$$0 \leq r \leq 1 \quad (23)$$

$$\alpha_{jo} \geq 0 \quad \forall j, o$$

$$\lambda_{ko} \geq 0 \quad \forall k, o$$

$$\delta_{ko} \text{ free } \forall k, o$$

The objective function (17) seeks to optimize the sum of the efficiencies for all of the DMUs, which are the weighted outputs in this MAOM model. In (18), the weighted outputs are restricted to be greater than or equal to a global efficiency variable  $r$ , which exists on  $[0, 1]$ . This constraint is important as one could imagine the objective function seeking to reduce the efficiency of one DMU to near zero in order to make the others nearer to one.

In (19), we force the sum of the weighted outputs to be less than or equal to the sum of the weighted inputs after adjusting them up or down by the amount necessary to achieve the highest sum of efficiency scores for each selected DMU ( $o=v$ ). This constraint applies weights generated for each separate DMU analysis to all other DMUs inputs and outputs for relative efficiency comparison, just as is done in traditional DEA. This constraint makes the problem non-linear since the input weights are multiplied against the input changes.

In (20), we force the sum of the weighted and adjusted inputs to be equal to one for each DMU. Doing so ensures that we will have efficiency scores for each DMU less than or equal to one. Again, this constraint is non-linear.

In (21), we force each remaining input (after adjustment) for each DMU to be greater than or equal to zero. Negative resources are not feasible.

The constraints in (22) require that any input adjustments sum to zero. We cannot grow resources for reallocation. Finally, the last set of constraints depicted in (23) is the bounds for the decision variables.

One might also include management constraints regarding the maximum movement of resources to increase flexibility and reflect management input into the system, similar to the  $f$  function provided by the MOLP. Doing so would simply require bounds on the appropriate  $\delta$ . These constraints would represent decision-maker input, similar to the development of the  $f$  function in multiple objective programming.

## **Chapter 4. Results and Discussion**

The solution to the MAOM program presented above provides the decision-maker recommendations regarding staffing of providers and allocation of funding such that all facilities achieve at least the efficiency associated with the  $r$  constraint. With this model formulation, there is a method for providing information regarding the adjustment of all inputs and outputs independent of or dependent upon decision-maker input.

Using the General Algebraic Modeling System (GAMS, 2013) as the modeling language and the CONOPT (Drud, 1992) and MINOS (Murtagh & Saunders, 1983) non-linear programming (NLP) solvers, a simple, hospital-based textbook problem (Anderson et al., 2012) was initially examined followed by a real-world example involving sixteen U.S. Army hospitals in the MHS with data from 2003. In the textbook example, seven different hospitals (DMUs) were initially evaluated using standard CRS DEA. The adjustable inputs for the hospitals included Full-Time Equivalents (FTEs), supply expenses in 1,000's, and available beds in 1,000's. Outputs include patient-days for those 65 and older in 1000's, patient-days for those under 65 in 1,000's, nurses trained, and interns trained. The data are shown in Table 1 below:

**Table 1.** Data from Textbook Example

DMU	FTEs	Supply Expenses	Available Bed Days	Patient Days $\geq 65$	Patient Days $< 65$	Nurses Trained	Interns Trained
A	310.0	134.6	116.0	55.31	49.52	291	47
B	278.5	114.3	106.8	37.64	55.63	156	3
C	165.6	131.3	65.52	32.91	25.77	141	26
D	250.0	316.0	94.4	33.53	41.99	160	21
E	206.4	151.2	102.1	32.48	55.3	157	82
F	384.0	217.0	153.7	48.78	81.92	285	92
G	530.1	770.8	215	58.41	119.7	111	89

Solving the problem using CCR DEA models results in all hospitals being efficient with the exception of Hospital D, which is 90.73% efficient. Reduced costs suggest that to enter the model, FTEs would need to be reduced by 12.16, expenses reduced by \$184.63K, and the number of interns adjusted by 7.67. The reference set for DMU D includes hospitals A, B, and E (meaning that the dual values are non-zero).

From typical sensitivity analysis, a conclusion might be to reduce resources for Hospital D. In this system, however, inputs are fixed. They may be spread across the hospital system but not cut (at least in the short-term). This leads us to using the MAOM formulation provided in (17) through (23), setting  $r$  (the minimum efficiency for any facility) to at least .95. Using the CONOPT non-linear solver in GAMS with two side constraints to prevent reallocation of more than 25%, a solution is reached nearly instantaneously. The resultant analysis provides efficiency scores equal to one for all facilities. No solution could be better, although other alternate solutions to the same problem do exist, and these alternatives will be more fully discussed in the second, real-world model. The input adjustment matrix provides recommendations for each DMU and each input that when re-implemented into the CCR DEA confirm all efficiency scores equal to one. The new values for the inputs follow in Table 2.

**Table 2.** Results from MAOM Using the Textbook Example

DMU	FTEs	Supply Expenses	Available Bed Days
A	310.10	134.60	117.171
B	278.50	114.30	108.143
C	165.60	131.30	64.936
D	250.00	316.00	86.563
E	206.40	151.20	103.060
F	384.00	217.00	153.702
G	530.10	770.80	219.946

The analysis suggests that by changing only available bed days for facilities (which means adding or removing beds), the efficiency scores might be improved the most. The optimality attained for this problem is only one of several optimal solutions available. For example, using the CONOPT NLP solver in GAMS resulted in an alternate (but similar) solution set. Detailing multiple solution sets that result in maximizing the objective function is necessary to provide decision support.

With this simple textbook example in hand, we move to the analysis of 16 Army MTFs with inputs and outputs that were deemed important to decision-makers in evaluating efficiency. The data are from 2003 (as to be non-sensitive in nature), and the hospitals were chosen from 24 facilities because they are largely homogenous. The inputs that could be manipulated included the funding stream (COST) – expenditures (in 1000s) less graduate medical education and readiness costs, and inflated in two parts to 2003 dollars – and the FTEs (FTE) – number of assigned full-time equivalents (in 1000s) in 2003. A non-discretionary input was the enrollment population supported (ENROLL) – enrollment population supported (in 1000s) in 2003. The outputs of interest included: inpatient aggregated MTF relative weighted product (in 1000s) in 2003 (RWP), outpatient aggregated MTF relative value units (in 1000s) in 2003 (RVU), a prevention/quality composite score found in MHS survey scaled between [0, 100] in 2003 (PREV), a satisfaction composite score found in MHS survey scaled between [0, 100] in 2003

(SAT), and an ease of access composite score found in MHS survey scaled between [0, 100] in 2003 (ACCESS). The original data are shown in Table 3.

**Table 3.** Data from Real-World Example (U.S. Army Hospitals)

	ENROLL	FTE	COST	RWP	RVU	PREV	ACCESS	SAT
H1	14.81	7.13	56.66	7.05	112.21	83.28	70.55	73.19
H2	23.09	9.86	72.67	6.51	182.38	83.40	66.24	71.55
H3	68.40	17.66	163.99	21.74	372.06	78.89	57.29	63.02
H4	80.62	17.20	169.14	14.14	476.48	89.14	67.39	73.63
H5	49.84	15.25	125.44	16.87	314.98	85.65	65.72	72.02
H6	38.13	13.04	130.23	10.41	229.08	84.82	65.61	69.87
H7	32.87	8.68	67.25	10.74	187.00	79.70	67.86	70.83
H8	12.74	6.34	53.16	7.07	85.10	84.60	67.49	73.67
H9	23.95	11.73	95.60	14.31	253.72	83.15	70.59	74.81
H10	14.93	6.42	52.37	0.96	76.53	89.44	65.40	69.85
H11	47.87	16.91	129.16	21.93	339.66	85.73	69.30	74.35
H12	31.50	8.81	71.98	3.01	153.56	82.32	60.92	69.81
H13	22.99	11.13	99.60	6.71	252.20	85.63	74.52	80.99
H14	31.39	12.73	92.53	14.87	298.59	83.97	70.48	75.62
H15	10.70	6.22	38.08	3.03	60.06	80.83	64.76	72.84
H16	63.40	14.71	114.29	14.86	327.31	80.24	62.89	68.53

For this more complex analysis, we ran variable returns to scale (VRS) DEA analysis, as such an analysis reasonably assumes that the production frontier is not necessarily linear. Assuming that enrollment is a non-discretionary input, facilities with inefficiency scores less than 1.0 included: H1 (.851), H2 (.928), H5 (.948), H7 (.779), H8 (.951), H9 (.850), H11 (.998), H13 (.959), H14 (.842), and H16 (.974).

Running the data in the MAOM using the CONOPT and MINOS solvers in GAMS resulted in all facilities achieving efficiency scores of 1.0 by changing funding (COST) and FTEs as shown in Table 4 (CONOPT) and in Table 5 (MINOS). Both NLP solvers executed this analysis (see the Appendix for the GAMS code) nearly instantaneously. What is interesting about the CONOPT results is that the changes, while significant, are not so severe as to require side

constraints. The MINOS solver, however, required two side constraints to prevent near elimination of FTEs for facility H12. This result shows the importance of binding the feasible region better, especially since multiple optima are possible if not likely. The constraints added for the MINOS Solver prevented more than 25% reductions or 25% increases in flexible inputs. (NOTE: it is even possible that the auto-optimization might have recommended the elimination of FTEs or funding from a facility).

**Table 4.** Results from MAOM Using Real-World Example and CONOPT

	ORIGINAL COST	NEW COST	ORIGINAL FTE	NEW FTE
H1	56.66	53.41	7.13	7.94
H2	72.67	72.63	9.86	9.64
H3	163.99	163.98	17.66	16.35
H4	169.14	174.81	17.20	17.47
H5	125.44	125.43	15.25	13.33
H6	130.23	130.21	13.04	10.28
H7	67.25	64.98	8.68	10.42
H8	53.16	53.16	6.34	6.16
H9	95.60	95.59	11.73	11.62
H10	52.37	52.38	6.42	6.30
H11	129.16	129.15	16.91	17.00
H12	71.98	71.92	8.81	7.56
H13	99.60	99.61	11.13	10.55
H14	92.53	92.57	12.73	19.33
H15	38.08	38.10	6.22	5.39
H16	114.29	114.25	14.71	14.48

**Table 5.** Results from MAOM Using Real-World Example and MINOS

	ORIGINAL COST	NEW COST	ORIGINAL FTE	NEW FTE
H1	56.66	56.66	7.13	8.912
H2	72.67	72.67	9.86	7.573
H3	163.99	163.99	17.66	17.66
H4	169.14	169.14	17.20	17.20
H5	125.44	125.44	15.25	13.848
H6	130.23	130.23	13.04	9.78
H7	67.25	67.25	8.68	10.850

H8	53.16	53.16	6.34	7.925
H9	95.60	95.60	11.73	11.730
H10	52.37	52.37	6.42	6.420
H11	129.16	129.16	16.91	18.814
H12	71.98	71.98	8.81	6.607
H13	99.60	99.60	11.13	11.13
H14	92.53	92.53	12.73	15.912
H15	38.08	38.08	6.22	6.22
H16	114.29	114.29	14.71	13.238

Again, we note that multiple solutions are likely to be available for many problems. Investigating these multiple optimal solutions is something that is important in order to provide quality decision-support. To investigate the optimal solution set, we next ran the MAOM using a multi-start genetic algorithm (GA) solver, MSNLP (Smith & Lasdon, 1992), and the side constraints specified for the MINOS runs. We allowed the GA to run for 1000 seconds and 1000 iterations. The GA solver found the first optimal in 200 iterations (during pre-processing). Afterwards, propagation continued for the full 1000 iterations. The final offspring with the best merit function resulted in the solution provided by Table 6; however, one of the real advantages to the multi-start GA approach is that it produces a family of possibilities for decision-makers to consider. We will discuss this later.

**Table 6.** Results from MAOM Using Real-World Example and MSNLP

	ORIGINAL COST	NEW COST	ORIGINAL FTE	NEW FTE
H1	56.66	42.509	7.13	8.912
H2	72.67	76.519	9.86	9.027
H3	163.99	140.318	17.66	18.614
H4	169.14	211.425	17.20	14.424
H5	125.44	121.896	15.25	15.496
H6	130.23	97.672	13.04	10.794
H7	67.25	75.117	8.68	10.792
H8	53.16	39.87	6.34	7.925
H9	95.60	119.5	11.73	11.609
H10	52.37	48.317	6.42	4.815
H11	129.16	96.87	16.91	21.137

H12	71.98	75.358	8.81	6.607
H13	99.60	117.944	11.13	8.428
H14	92.53	97.419	12.73	15.912
H15	38.08	28.56	6.22	6.221
H16	114.29	142.857	14.71	13.106

We notice that CONOPT recommended minor changes in funding, while MINOS recommended no changes (a constant). MSNLP, however, produced more significant funding shifts. For FTEs, we noticed that the directionality change of 11 of the 16 models was identical. Table 7 provides a congruency analysis based on recommended adjustments by the three solvers.

**Table 7.** Congruency Analysis of Solvers for FTEs

	MINOS	CONOPT	MSNLP	CONGRUENCE
H1	0.81	1.782	1.782	Positive
H2	-0.22	-2.287	-0.833	Negative
H3	-1.31	0	0.954	Mixed
H4	0.27	0	-2.776	Mixed
H5	-1.92	-1.402	0.246	Mixed
H6	-2.76	-3.26	-2.246	Negative
H7	1.74	2.17	2.112	Positive
H8	-0.18	1.585	1.585	Mixed
H9	-0.11	0	-0.121	Semi-Negative
H10	-0.12	0	-1.605	Semi-Negative
H11	0.09	1.904	4.227	Positive
H12	-1.25	-2.203	-2.203	Negative
H13	-0.58	0	-2.702	Semi-Negative
H14	6.6	3.182	3.182	Positive
H15	-0.83	0	0.001	Mixed
H16	-0.23	-1.472	-1.604	Negative

The results imply that a set of possible solutions should be presented to decision-makers considering reallocation of inputs, and the use of multi-start evolutionary algorithms appears to be a reasonable method for doing so. In fact, quick congruence may be desirable; however, if any side constraints are missing, the results may be less than optimal from a decision-makers' perspective. We underscore the value of additional constraints in analyses which are likely to



produce multiple solutions. We also emphasize the value of investigating a family of solutions that maximize the efficiency of the overall system, as a single decision set may not provide sufficient flexibility for decision makers.

## **Chapter 5. Conclusions**

By exploring the similarities between optimization methods for handling multiple objective problems, a related non-linear, multi-objective, resource allocation-based optimization program that allows for the adjustment of resources (system inputs) either with or without decision-maker input was generated, programmed and solved on a representative data set. The utility for this type of decision-support model to be employed in support of resource allocations for large, centrally funded hospital systems is self-evident. As demand for health services increases, the need for efficient allocation models based on competing objectives will become increasingly more important, and models similar to those proffered here will aid decision-makers' efforts.

In conducting this analysis, we found utility in the use of multi-start evolutionary algorithms that store multiple optimal solutions for consideration by decision-makers. We also found that the addition of appropriate side constraints could ameliorate deviations of significant magnitude. Future work will see the implementation of these methods on the entire MHS with expanded inputs and outputs coupled with panel data

## Appendix – GAMS Code

\$ontext

Auto-optimization

\$offtext

\$offlisting

sets

o	DMUs or hospitals in this analysis /H1*H16/
ds(o)	DMU set for reporting
n	index set /ENROLL, FTE, COST, RWP, RVU, PREV, ACCESS, SAT /
j(n)	output index /RWP, RVU, PREV, ACCESS, SAT/
k(n)	input index /ENROLL, FTE, COST/

Parameter

data(o,n)	unit input output
OptFile 1	

;

Variables

d(o,k)	resource change
a(o,j)	output weights
l(o,k)	input weights
r(o)	efficiency score minimum
z	objective
eff(o)	efficiency
var	convexity for BCC

positive variables a(o,j), l(o,k), r, eff(o);

free variables d(o,k),z;

Equations

superobj	superobjective
obj(o)	objective function
c1(o)	minimum efficiency
c2a(o)	outputs less than or equal to inputs with weights from DMU 1
c2b(o)	outputs less than or equal to inputs with weights from DMU 2
c2c(o)	outputs less than or equal to inputs with weights from DMU 3
c2d(o)	outputs less than or equal to inputs ...
c2e(o)	outputs less than or equal to inputs
c2f(o)	outputs less than or equal to inputs
c2g(o)	outputs less than or equal to inputs
c2h(o)	outputs less than or equal to inputs
c2i(o)	outputs less than or equal to inputs

c2j(o)	outputs less than or equal to inputs
c2k(o)	outputs less than or equal to inputs
c2l(o)	outputs less than or equal to inputs
c2m(o)	outputs less than or equal to inputs
c2n(o)	outputs less than or equal to inputs
c2o(o)	outputs less than or equal to inputs
c2p(o)	outputs less than or equal to inputs
c3(o)	adjust input resources
c4(o,k)	adjusted inputs must be non-negative
c5(k)	all adjustments must sum to zero
c6(o)	minimum efficiency must be greater than lower bound
c7(o)	maximum efficiency must
c8(o,k)	enrollment is non-discretionary
c9(o,k)	side constraints preventing rapid decreases or increases of inputs MINOS
c10(o,k)	side constraints preventing rapid decreases or increases of inputs MINOS

;

superobj.. z=e=sum(o,eff(o));

obj(o).. eff(o)=e=sum((j),a(o,j)\*data(o,j))-l\*var;

c1(o).. sum(j,a(o,j)\*data(o,j))=g=r(o);

c2a(o).. sum(j,a('H1',j)\*data(o,j))-sum(k,l('H1',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2b(o).. sum(j,a('H2',j)\*data(o,j))-sum(k,l('H2',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2c(o).. sum(j,a('H3',j)\*data(o,j))-sum(k,l('H3',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2d(o).. sum(j,a('H4',j)\*data(o,j))-sum(k,l('H4',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2e(o).. sum(j,a('H5',j)\*data(o,j))-sum(k,l('H5',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2f(o).. sum(j,a('H6',j)\*data(o,j))-sum(k,l('H6',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2g(o).. sum(j,a('H7',j)\*data(o,j))-sum(k,l('H7',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2h(o).. sum(j,a('H8',j)\*data(o,j))-sum(k,l('H8',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2i(o).. sum(j,a('H9',j)\*data(o,j))-sum(k,l('H9',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2j(o).. sum(j,a('H10',j)\*data(o,j))-sum(k,l('H10',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2k(o).. sum(j,a('H11',j)\*data(o,j))-sum(k,l('H11',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2l(o).. sum(j,a('H12',j)\*data(o,j))-sum(k,l('H12',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2m(o).. sum(j,a('H13',j)\*data(o,j))-sum(k,l('H13',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2n(o).. sum(j,a('H14',j)\*data(o,j))-sum(k,l('H14',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2o(o).. sum(j,a('H15',j)\*data(o,j))-sum(k,l('H15',k)\*(data(o,k)+d(o,k)))-var=l=0;

c2p(o).. sum(j,a('H16',j)\*data(o,j))-sum(k,l('H16',k)\*(data(o,k)+d(o,k)))-var=l=0;

c3(o).. sum(k,l(o,k)\*(data(o,k)+d(o,k)))=e=1;

c4(o,k).. data(o,k)+d(o,k)=g=0;

c5(k).. sum(o,d(o,k))=e=0;

c6(o).. r(o)=g=.90;

c7(o).. r(o)=l=1.0000000000001;

c8(o,k).. d(o,'ENROLL')=e=0;

c9(o,k).. d(o,k)=g=-.25\*data(o,k);

c10(o,k).. d(o,k)=l=-.25\*data(o,k);

\*The model includes all constraints

model autoopt autoopt /superobj,obj,c1,c2a,  
c2b,c2c,c2d,c2e,c2f,c2g,c2h,c2i,c2j,c2k,c2l,c2m,c2n,c2o,c2p,c3,c4,c5,c6,c7,c8,c9,c10/  
;

\*To run military data, name table "data". Otherwise name "data2."

\*MED=RWPs, NONMED=RVUs, RN=PREV, INTERN=Sat

Table data(o,n)

	ENROLL	FTE	COST	RWP	RVU	PREV	ACCESS
SAT							
H1	14.81	7.13	56.66	7.05	112.21	83.28	70.55
H2	23.09	9.86	72.67	6.51	182.38	83.40	66.24
H3	68.40	17.66	163.99	21.74	372.06	78.89	57.29
H4	80.62	17.20	169.14	14.14	476.48	89.14	67.39
H5	49.84	15.25	125.44	16.87	314.98	85.65	65.72
H6	38.13	13.04	130.23	10.41	229.08	84.82	65.61
H7	32.87	8.68	67.25	10.74	187.00	79.70	67.86
H8	12.74	6.34	53.16	7.07	85.10	84.60	67.49
H9	23.95	11.73	95.60	14.31	253.72	83.15	70.59
H10	14.93	6.42	52.37	0.96	76.53	89.44	65.40
H11	47.87	16.91	129.16	21.93	339.66	85.73	69.30
H12	31.50	8.81	71.98	3.01	153.56	82.32	60.92
H13	22.99	11.13	99.60	6.71	252.20	85.63	74.52
H14	31.39	12.73	92.53	14.87	298.59	83.97	70.48
H15	10.70	6.22	38.08	3.03	60.06	80.83	64.76
H16	63.40	14.71	114.29	14.86	327.31	80.24	62.89

\$eolcom //

;

\*var.fx = 0; // to run CRS with the primal model

var.lo = -inf; // to run VRS with the primal model

var.up = +inf; // to run VRS with the primal model

```
*option nlp=conopt;  
*option nlp=minos;  
option nlp=msnlp;  
autoopt.OptFile=1;
```

\*We see to maximize our objective function.  
solve autoopt using nlp max z ;

\*We specify a set for producing reports.  
set ii(o) /H1\*H16/

\*We define two reports.

```
parameter rep Summary Report;  
parameter rep2 Summary Report;
```

```
loop (ii, ds(ii)=yes;  
*rep(ii, 'ENROLL '=)=data(ii,'ENROLL');  
*rep(ii,'NEW ENROLL '=)=d.l(ii,'ENROLL')+data(ii,'ENROLL');  
rep(ii,'COST '=)=data(ii,'COST');  
rep(ii,'NEW COST '=)=d.l(ii,'COST')+data(ii,'COST');  
rep(ii,'FTE '=)=data(ii,'FTE');  
rep(ii,'NEW FTE '=)=d.l(ii,'FTE')+data(ii,'FTE');  
ds(ii)=no);  
rep2('The Obj Function '=) = autoopt.objval;
```

```
display rep;  
display rep2;
```

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2008 – 2009: **M.Sc.**, *Econometrics & Operations Research*, Department of Quantitative Economics, School of Business and Economics, Maastricht University, Maastricht, NL.

2004 – 2008: **B.S.**, *Engineering Management w/ Honors*, Department of Systems Engineering, United States Military Academy, West Point, NY.

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2008 – Present: *Associate Engineering Manager*, Engineering Manager Professional Certification Program, American Society for Engineering Management

2008 – Present: *Engineer-in-Training (Intern Engineer) – Industrial Engineering*, National Council of Examiners for Engineering and Surveying Fundamentals of Engineering Certification, The University of the State of New York Professional Engineering & Land Surveying Education Department, Certificate #088473

#### TEACHING APPOINTMENTS AND EXPERIENCE

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2013: *Guest Lecturer*, Department of Computer Information Systems & Quantitative Methods, McCoy College of Business Administration, Texas State University, San Marcos, TX.

2013 – Present: *Adjunct Instructor of Quantitative Methods*, United States Army – Baylor University, Graduate Program in Health and Business Administration, Fort Sam Houston, TX.

2012 – 2013: *Adjunct Instructor of Mathematics*, Department of Mathematics and Statistics, The Undergraduate School, University of Maryland University College, Kandahar, AF.

#### HONORS AND AWARDS (MOST NOTABLE)

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2013: INFORMS-Seth Bonder Scholarship for Applied Operations Research in Military Applications

2012: Center for Integrated Healthcare Delivery Systems (CIHDS) Scholarship

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2008: The Netherland-America Foundation (NAF) Fulbright Philips Electronics Fellowship

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